Last Updated:
Spring 1993

NUMERICAL ANALYSIS TOPICS LIST

A. Matrix Theory

1. Solution of linear equations: Guassian elimination, scaling, pivoting (stability), condition number, iterative methods, least squares.
2. Eigensystem calculations: Simple perturbation theory, Gershgorin theorem, power method, Rayleigh quotient, Householder transformation, QR decomposition, QR method for calculating eigenvaules.

References:

1. G.W. Stewart: *Introduction to Matrix Computations*, Academic Press (1973).
2. J.H. Wilkinson: *The Algebraic Eigenvalue Problem*, Clarendon Press, Oxford (1965).

B. Quadrature

1. Order of error, convergence properties.
2. Gauss quadrature, Newton-Cotes quadrature.

References: See supplementary texts below.

C. Approximation Theory

1. General results: Abstract theory, existence of a best approximation in a finite-dimensional subspace of a Banach space, uniqueness, characterization.
2. Examples: Least squares, least-mean, and uniform (or Chebyshev) approximation.
3. Approximation families: Polynomials, trigonometric polynomials, splines.
4. Degree of approximation: Weierstrass Theorem, Jackson Theorem.
5. Good approximation: Linear methods: interpolation, least-squares approximation, projection, Chebyshev polynomials, Bernstein polynomials.

References:

1. E.W Cheney: *Introduction to Approximation Theory* McGraw-Hill (1966)
2. C. de Boor: *A Practical Guide to Splines*, Springer (1978).

D. Functional Analysis

1. Basic Banach space theory.
2. Contraction mapping theorem.
3. Uniform boundedness theorem.
4. Iterative methods (with application to nonlinear algebraic systems).
5. Newton's method.

References:

1. C. Groetsch: *Elements of Applicable Functional Analysis*, M. Dekker (1980).
2. P. Linz: *Theoretical Numerical Analysis*, Wiley (1979).

3. C. de Boor: *Numerical Functional Analysis*, Lecture Notes for CS 717, Fall 1991.

E. Ordinary Differential Equations

1. Initial value problems: Single step methods, multistep methods, predictor-corrector methods, stability, convergence, "stiff" equations.
2. Boundary value problems: Shooting methods, difference methods, deferred correction, newton's method, uniqueness.

References:

1. H.B. Keller: *Numerical Methods for Two-Point Boundary Value Problems*, Blaisdel (1968).
2. J. D. Lambert: Computational Methods on O.D.E., Wiley (1973).
3. C. W. Gear: *Numerical Initial Value Problems in Ordinary Differential Equations*, Prentice-Hall (1971).
4. S. V. Parter: *Numerical Analysis of Differential Equations*, Notes from CS 713, Spring, 1992.

F. Partial Differential Equations

1. Initial value problems: Hyperbolic and parabolic equations, stability, consistency, convergence, von Neumann analysis for stability, dissipation, dispersion.
2. Boundary value problems: Finite difference methods, maximum principle, finite element methods, variational methods, existence and uniqueness of solutions.

References:

1. J. C. Strikwerda, *Finite Difference Schemes and Partial Differential Equations*, Wadsworth/Brooks-Cole (1989).
2. G. E. Forsythe and W. R. Wasow: *Finite Difference Methods for P.D.E.*, Wiley (1960).
3. R. D. Richtmyer and K. W. Morton: *Difference Methods for Initial-Value Problems*, 2nd Ed. Interscience (1967) Chapters 1-4, 8.
4. G. Strang and G. Fix: *An Analysis of the Finite Element Method*, Prentice-Hall (1973).

Supplementary Texts:

1. G. Dahlquist and A. Bjorck: *Numerical Methods*, Prentice-Hall (1974).
2. E. Isaacson and H. B. Keller: *Analysis of Numerical Methods*, Wiley (1966).
3. K. Atkinson: *An Introduction to Numerical Analysis*, Wiley (1978).
4. S. D. Conte and C. de Boor: *Elementary Numerical Analysis*, 3rd Ed., McGraw-Hill (1980).

The basic courses 513 and 514 cover most of the material in A, B, C, and E and are prerequisite for the breadth exam. The depth exam is base in addition on the courses 712, 713, 717 which cover the material in F, E, and D respectively.