What kinds of pieces?

- Line segments
- Low-order polynomials
  - Quadrics (degree 2)
  - Cubics (degree 3)
  - Quartics, quintics, …
- Cubics are most popular in graphics
  - Best balance

What to control

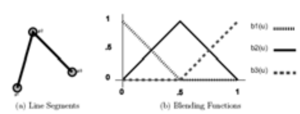
- Control points
  - Where a curve goes (at a particular parameter value)
  - Derivaive (at a particular parameter value)
- Specify values at a site
- Specify line segment
  - End points
  - Center and one end
  - Center and offset to end
  - Center, length, orientation (non-linear change)

Polynomial Interpolation

- A degree d polynomial can interpolate d+1 points
  - Line segments = 2
  - Quadrics = 3
  - Cubics = 4
  - …
- But…
  - Hard to control
  - Non-Local
  - Hard to compute accurately
- So, we rarely use high-order polynomials in graphics

Line segments

- Endpoints $\mathbf{p_1}$ and $\mathbf{p_2}$
  - $\mathbf{p} = (1-u)\ \mathbf{p_1} + u\ \mathbf{p_2}$
- Blending functions
  - $\mathbf{p} = b_1(u)\ \mathbf{p_1} + b_2(u)\ \mathbf{p_2}$
  - Convenient way to describe functions (including polynomials)
  - Basis functions (scalar functions)



Line Segment Bases

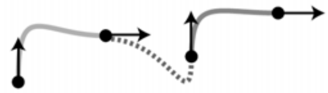
- Could choose different controls for line segment
  - Whatever was convenient
- Find conversions between different representations

Cubics

- Different than book: explain cubic forms first, derive them second
- Canonical form for polynomial
  - $f(u) = \sum a_i u^i$
  - Vector a of coefficients
- Polynomial coefficients not very convenient
  - $a_3 u^3 + a_2 u^2 + a_1 u + a_0$

Different ways to describe a cubic

- Positions of 4 points
  - u = 0, 1/3, 2/3, 1
  - Easy for 1 segment, hard to make connections
- Position & derivative at beginning and end
  - Hermite form

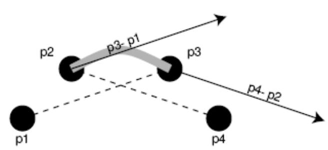


More ways to describe cubics

- Natural Cubics
  - "smoothest" curve
  - C(2)
- Each piece:
  - u=0: position, 1st derivative, 2nd derivative
  - u=1: position
- Piece 2 looks at values of previous piece (at end)
  - Propagation
- Non-local control (change at beginning changes everything)

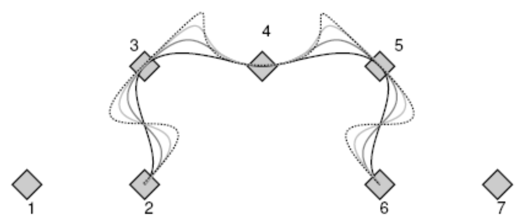
Cardinal Cubics
Catmull-Rom Splines

- Interpolate points
- Each segment interpolates p_1 and p_2
  - u=0, p_1 – derivative is k (p_2 – p_0)
  - u=1, p_2 – derivative is k (p_3 – p_1)
  - K = ½ for Catmull-Rom



Cardinal Interpolation

- k = ½ (1-t)   t = tension (0 for Catmull-Rom)



Approximating Curves

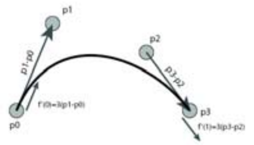
- Interpolation isn't the only way to describe a curve
- Give points that "influence" a curve
- Why?
  - Better control of what happens in between points
- 2 important cases for computer graphics
  - Bezier
  - B-Spline

Bezier Segments

- Curve is made of many segments
  - Nomenclature issue
- Each segment is a polynomial
  - Of any degree
  - 3 is most common in computer graphics

Bezier Segments

- A segment of degree d has (d+1) control points
- A segment interpolates its first and last controls
  - With u= 0, 1 respectively
- The first derivative at the beginning (end) is proportional to the vector between the first 2 (last 2) points – scaled by the degree of the curve



Bezier Segments (2)

- The nth derivative depends on the first (or last) n points
- Cubics are similar to Hermites
  - All points in space (not derivative amounts)
  - Scaling factors
- Pieces connected by placing points correctly
  - C(0) by matching endpoints
  - C(1) by aligning end vectors
  - G(1) by end-vectors being co-linear

Properties of Bezier Curves

- Simple mathematical form for basis functions
- Good algorithms for computation
  - Subdivision procedure
  - De Casteljau algorithm
  - Divide and conquer because…
- Convex Hull Properties
- Variation Diminishing
- Symmetric
- Affine invariant
  - NOT perspective invariant